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Frontier Estimation and Firm-Specific Inefficiency Measures in the Presence of Heteroscedasticity

Steven B. Caudill
Department of Economics, Auburn University, Auburn, AL 36849-5242

Jon M. Ford
Department of Economics, Florida Atlantic University, Boca Raton, FL 33432

Daniel M. Gropper
Department of Economics, Auburn University, Auburn, AL 36849-5242

The purpose of this article is to illustrate a straightforward and useful method for addressing the problem of heteroscedasticity in the estimation of frontiers. A heteroscedastic cost-frontier model is developed and estimated using bank cost data similar to that used by Ferrier and Lovell. Our results show dramatic changes in the estimated cost frontier and in the inefficiency measures when accounting for heteroscedasticity in the estimation process. We find that the rankings of firms by their inefficiency measures is affected markedly by the correction for heteroscedasticity but not by alternative distributional assumptions about the one-sided error term.

KEY WORDS: Bank costs; Heteroscedastic inefficiency; Stochastic frontier.

Since its introduction by Aigner, Lovell, and Schmidt (1977), stochastic frontier estimation has been widely used in empirical work. Recent applications include the estimation of frictional unemployment by Warren (1991), the estimation of earnings functions by Polachek and Yoon (1987), the estimation of bank efficiency by Ferrier and Lovell (1990), the estimation of farm efficiency in Kansas by Thompson, Langemeier, and Lee (1990), the estimation of electric-utility efficiency by Reifschneider and Stevenson (1991), and the estimation of the efficiency of life-insurance firms by Yuengert (1993).

The measures of inefficiency used in these studies are based on residuals obtained from the estimation of a frontier. Residuals are sensitive to specification errors, particularly in frontier models, and it is likely that this sensitivity will be passed on to the inefficiency measures. Heteroscedasticity is a specification error often associated with the estimation of cost functions, and the presence of heteroscedasticity is likely to affect these inefficiency measures. It is well known that heteroscedasticity does not have much harmful effect on estimators of average practice cost functions—estimators remain unbiased but are no longer efficient. This occurs because average practice cost functions are usually estimated by least squares, which yields a mean regression, and means are not affected by symmetric dispersions around them. The problem of heteroscedasticity is far more serious in frontier models because, unlike the mean regression function, the frontier *is* changed when the dispersion increases.

This article illustrates how to estimate a heteroscedastic frontier cost function. We are motivated to develop a heteroscedastic frontier model by the results of a Monte Carlo study by Caudill and Ford (1993) and by the results of our own Monte Carlo study, which are available on request. Caudill and Ford investigated the effects of heteroscedasticity in the one-sided error on parameter estimates in a single-factor frontier production function. They found that heteroscedasticity leads to biased parameter estimates. Specifically, when the model is estimated by maximum likelihood, heteroscedasticity leads to overestimation of the intercept and underestimation of the slope coefficients. (The opposite should be the case for a cost frontier.) Our own Monte Carlo study confirmed the findings of Caudill and Ford and went further to determine the effects of heteroscedasticity in the one-sided error on some commonly used inefficiency measures. Not surprisingly, the inefficiency measures are also affected by the heteroscedasticity. In our Monte Carlo study of the estimation of a cost frontier, not accounting for the heteroscedasticity in the estimation led to the overestimation of inefficiency for small firms and the underestimation of inefficiency for large firms. For these reasons we believe that the development and estimation of a heteroscedastic frontier model is an important task.

In this article we develop and estimate a heteroscedastic frontier cost function using data from commercial banks. Our results show dramatic changes in the parameter estimates and inefficiency measures when accounting for heteroscedasticity in the estimation process. We find that the rankings of firms by their inefficiency measures is affected markedly by correcting for heteroscedasticity but not by alternative assumptions about the distribution of the one-sided error term. Thus the changes in firm-specific inefficiency measures in both absolute and relative terms indicate that those concerned about the measurement of firm-specific inefficiency should consider testing for and, if indicated, correcting for heteroscedasticity in their estimation procedures.

1. FRONTIER ESTIMATION AND INEFFICIENCY MEASURES

Frontier techniques have been widely applied to the estimation of both production and cost functions, and without loss of generality we confine our discussion to the estimation of cost frontiers. Prior to the article by Aigner et al. (1977), primarily average practice cost functions were estimated, usually by ordinary least squares (OLS). The models were specified as

$$\mathrm{TC}_i = X_i\beta + w_i, \qquad (1)$$

where TC_i is total cost, X_i is a vector of explanatory variables including output quantities and input prices, β is a vector of unknown parameters to be estimated, and w_i is a two-sided error term with $E(w_i) = 0, E(w_iw_j) = 0$ for all i and j, $i \neq j$, and $V(w_i) = \sigma_w^2$.

Stochastic frontier estimation is based on the idea that some firms are less efficient than others and that deviations from the frontier are due to these inefficiencies. To represent this inefficiency in the cost relationship, an additional one-sided error term is added to the model in (1) so that it becomes

$$\mathrm{TC}_i = X_i\beta + w_i + v_i, \qquad (2)$$

where $E(v_i) > 0, E(v_iv_j) = 0$ for all i and $j, i \neq j$, and $V(v_i) = \sigma_v^2$. The assumption is also made that w and v are uncorrelated. The two-sided error term, w, is associated with things outside firm control, and the one-sided error term, v, is associated with factors under control of the firm. The importance of this distinction is made clear later.

The most common distributional assumptions made in frontier estimation are that w_i is normal and that v_i is either half-normal or exponential. First, we assume normal and half-normal, respectively. The density function of the sum of a normal and a half-normal was first derived by Weinstein (1964). Letting $\epsilon = w + v$, the density function is

$$f(\epsilon) = (2/\sigma)f^*(\epsilon/\sigma)F^*(\lambda\epsilon/\sigma), -\infty < \epsilon < +\infty, \qquad (3)$$

where $\sigma^2 = \sigma_w^2 + \sigma_v^2, \lambda = \sigma_v/\sigma_w$, and $f^*(\cdot)$ and $F^*(\cdot)$ are the standard normal density and distribution functions, respectively. The parameters of this model are β, σ, and λ, and the estimation of this model by maximum likelihood is routine.

One major advantage of stochastic frontier estimation is that it allows for the measurement of inefficiency. Aigner et al. (1977) and Schmidt and Lovell (1979) suggested measuring average inefficiency by $\sigma_v\sqrt{2}/\pi$. Alternatively, average inefficiency could be estimated by the average of the residuals, ϵ_i. The real advantage of frontier estimation, however, is that it permits the estimation of firm-specific inefficiency. Jondrow, Lovell, Materov, and Schmidt (1982) gave two measures of firm-specific inefficiency, both of which are based on the conditional distribution of v_i given ϵ_i. The first measure is based on the conditional expected value of v given ϵ and is given by

$$E(v \mid \epsilon) = \sigma_*[(\epsilon\lambda/\sigma) + (f^*(\epsilon\lambda/\sigma)/F^*(\epsilon\lambda/\sigma))], \qquad (4)$$

where $\sigma_* = (\sigma_v\sigma_w/\sigma)^2$ and all other expressions are as previously defined. Jondrow et al. also provided a measure of firm-specific inefficiency based on the conditional mode, which is given by

$$\begin{aligned} M(v \mid \epsilon) &= \epsilon(\sigma_v^2/\sigma^2) \quad &&\text{if } \epsilon \geq 0 \\ &= 0 &&\text{if } \epsilon < 0. \end{aligned} \qquad (5)$$

Both of these measures are easy to calculate, and they have been used extensively in production and cost studies, even though they are not consistent.

There is a problem associated with using measures based on residuals to make statements about firm-specific inefficiency. The problem is that residuals are particularly sensitive to specification errors. This problem was mentioned in a recent article on cost efficiency in banking by Ferrier and Lovell (1990), who stated, "The econometric approach imposes parametric structure on both technology and the distribution of inefficiency, and so commingles specification error with inefficiency" (p. 243).

Heteroscedasticity is one specification error that researchers can reasonably expect to encounter in the estimation of stochastic frontiers, either production or cost functions. In many econometrics textbooks readers are advised to expect heteroscedasticity when the observations are of "different size." This advice has its historical roots in the research of Prais and Houthakker (1955), who found expenditures for households with higher incomes to be more volatile than expenditures for households with lower incomes. This is true because people with higher incomes have more scope for choice. The same should be true of firms in their production and cost relationships. To be consistent with the traditional view that associates size-related heteroscedasticity with greater ability to choose, we consider the possibility that the one-sided error, v, is heteroscedastic. This one-sided error term embodies factors "under firm control" and larger firms have more "under their control."

Although the presence of heteroscedasticity has little harmful effect on the parameter estimates of average practice cost functions (estimators remain unbiased but are no longer efficient), the effects of heteroscedasticity on parameter estimates in frontier models are more troublesome. The Monte Carlo results of Caudill and Ford (1993) show that heteroscedasticity in the one-sided error term leads to biases when frontier models are estimated. The effect of heteroscedasticity on firm-specific inefficiency measures is also of considerable interest. Our own Monte Carlo results on the estimation of a frontier cost function show that heteroscedasticity in the one-sided error causes the intercept to be underestimated and the slope parameters to be overestimated. If the heteroscedasticity is related to the firm size, small firms appear to be less efficient and large firms more efficient if the heteroscedasticity is not considered in the estimation.

2. A HETEROSCEDASTIC FRONTIER MODEL

The estimation of heteroscedastic frontier models has been recently addressed in the literature by Reifschneider and Stevenson (1991) and Yuengert (1993). These authors took very different approaches to the incorporation of heteroscedasticity in frontier models, and both approaches differ from the approach taken here.

Reifschneider and Stevenson (1991) were the first to incorporate heteroscedasticity into a frontier model in their investigation of firm-specific inefficiency in the electric-utility industry. They incorporated heteroscedasticity into the composite error, ϵ, by allowing the mean of the one-sided error to vary. This is achieved by simply adding a constant, which can vary from firm to firm, to the usual one-sided error term. This flexibility permits the measurement of firm-specific inefficiency. In their article, Reifschneider and Stevenson anticipated the extension we have made in this article—that is, incorporating the heteroscedasticity directly into the variance of the one-sided error—but opted for their method because of its relative computational ease.

Recently, Yuengert (1993) incorporated heteroscedasticity into a frontier model using data on insurance firms. In his model, the two-sided error is normally distributed and the one-sided error is gamma distributed. Heteroscedasticity is incorporated directly into expressions for both variances; however, the variances are only allowed to assume seven different values corresponding to different classes according to asset size. Unlike Reifschneider and Stevenson and the method presented here, the variances are not firm-specific but group-specific.

To begin our discussion of the incorporation of size-related heteroscedasticity into frontier models, some assumption must be made as to the nature of the heteroscedasticity. Here we assume that the one-sided error exhibits what Greene (1990) referred to as multiplicative heteroscedasticity. Thus σ_v can be written

$$\sigma_{vi} = \sigma \exp(Z_i\gamma), \tag{6}$$

where Z_i is a vector of variables related to firm size and γ is a vector of unknown parameters. If Z_i includes an intercept, the preceding expression can be simplified to

$$\sigma_{vi} = \exp(Z_i\gamma). \tag{7}$$

This functional form has several advantages over others. Multiplicative heteroscedasticity has some computational advantages because it automatically constrains $\sigma_{vi} > 0$ and its use does not require division (which complicates numerical optimization). In addition, the functional form in (7) is easily constrained to yield the homoscedastic case, thus making a likelihood ratio test possible. In any case, Kennedy (1985) stated that Monte Carlo evidence suggests that, at least in linear models, precise knowledge of the functional form of the heteroscedasticity is not crucial to improving the estimation. We expect that this result is at least partially true for nonlinear models as well, and in this spirit the multiplicative functional form is adopted here.

To allow for heteroscedasticity in the frontier estimation, the model is parameterized in terms of β, σ_v, and σ_w. The standard deviation of the two-sided error term is also written in exponential form so that $\sigma_w = \exp(\theta)$. The density function in (3) can now be written

$$f_i(\epsilon_i) = (2/\sigma_i)f^*(\epsilon_i/\sigma_i)F^*(\lambda_i\epsilon_i/\sigma_i), \quad -\infty < \epsilon_i < +\infty, \tag{8}$$

where $\sigma_i^2 = \sigma_w^2 + \sigma_{vi}^2$, $\lambda_i = \sigma_{vi}/\sigma_w$, and f^* and F^* are as defined previously. Note that λ_i is a function of γ and that σ_i depends on λ_i, γ, and θ. The likelihood function can be written as the product of density functions like those in (8). The log-likelihood function is

$$\log L(\beta, \gamma, \theta) = \Sigma \log(f_i(\epsilon_i)). \tag{9}$$

Taking partial derivatives yields

$$\frac{\partial \log L}{\partial \beta} = \Sigma\left[\frac{(y_i - X_i\beta)}{\sigma_i^2} + \frac{\lambda_i f_i^*}{\sigma_i F_i^*}\right]X_i', \tag{10a}$$

$$\frac{\partial \log L}{\partial \gamma} = \Sigma\left[-\frac{\sigma_{vi}^2}{\sigma_i^2} + \frac{f_i^*}{F_i^*}\left(\frac{(y_i - X_i\beta)\lambda_i}{\sigma_i}\right) \times \left(-\frac{\sigma_{vi}^2}{\sigma_i^2} + 1\right) + \frac{\sigma_{vi}^2}{\sigma_i^2}\left(\frac{y_i - X_i\beta}{\sigma_i}\right)^2\right]Z_i', \tag{10b}$$

and

$$\frac{\partial \log L}{\partial \theta} = \Sigma\left[-\frac{\sigma_w^2}{\sigma_i^2} + \frac{f_i^*}{F_i^*}\left(\frac{(y_i - X_i\beta)\lambda_i}{\sigma_i}\right) \times \left(-\frac{\sigma_w^2}{\sigma_i^2} + 1\right) + \frac{\sigma_w^2}{\sigma_i^2}\left(\frac{y_i - X_i\beta}{\sigma_i}\right)^2\right]. \tag{10c}$$

Maximization of the likelihood function is accomplished by using the algorithm described by Berndt, Hall, Hall, and Hausman (1974).

3. AN APPLICATION

To illustrate the effects of heteroscedasticity in empirical research, the method presented in this article is applied to an interesting research problem. Data from the Federal Reserve System's Functional Cost Analysis (FCA) program were obtained. This data set has been used widely in previous research on the banking industry and in particular in frontier cost function estimation by Ferrier and Lovell (1990). It contains detailed information on the inputs used by participating institutions, as well as the number and dollar volume of the loans and deposit accounts at these institutions. A functional allocation of some costs across the institution's activities is made by the reporting institution's employees, although this allocation is not used in this study. The advantages of using this data for the present study are that detailed information is provided on bank inputs and outputs and its use enables comparison with previously published research. One disadvantage is that the data made available to the public has some information masked to protect the confidentiality of the participating institutions; for this reason the location of the firm is not revealed, and neither is holding-company affiliation.

Our objective in using this data set is not to reproduce Ferrier and Lovell's results. We use their published model and specification so that we can focus on the merits of estimating a heteroscedastic frontier rather than discuss particular variable definitions and model specifications.

To illustrate the problems inherent in frontier estimation in the presence of heteroscedasticity, we use FCA data from 1984 and define the variables as listed later, following the procedures of Ferrier and Lovell (1990). Five outputs are used, including the number of demand deposit accounts (y_1), the number of time deposit accounts (y_2), the number of real-estate loans (y_3), the number of installment loans (y_4), and the number of commercial loans (y_5). Three input prices were used—an average price of labor services (w_1), an approximate price of physical capital and equipment (w_2), and an approximate price of materials (w_3). The average price of labor services is calculated by dividing total expenditures on salaries and fringe benefits by the total number of bank employees. The approximation to the true price of physical capital services is calculated by dividing occupancy costs and spending on furniture and equipment by the dollar volume of deposits. Similarly, the approximation to the price of materials is calculated by dividing spending on materials by the dollar volume of deposits. These last two variables follow the procedures outlined by Mester (1987) and used by Ferrier and Lovell (1990); dividing by deposits is done because of the institutional age-related bias introduced if one divides physical capital spending by the book value of total physical capital, because physical capital is recorded at historical cost rather than market value. There is no quantity measure of material inputs available to convert total spending to a unit-price measure directly, so the dollar volume of total deposits is used on the basis that this amount of spending is used to support the reported volume of deposits. As Ferrier and Lovell noted, the definitions of outputs and input prices are less than perfect but are necessary because better information is not available, particularly on input prices. Linear homogeneity in input prices is imposed in the estimation.

The group of variables included to control for differences in bank costs due to factors other than input prices or output quantities again follows the procedures of Ferrier and Lovell (1990). Consistent with what has been termed the production approach to investigating bank costs, the number of loans and deposit accounts are used as a measure of bank output and the average dollar volume ($d1$–$d5$) of each type of loan or deposit is included to control for size-related differences in production costs [see Clark (1988), for a discussion of this and other approaches to measuring bank costs]. Other control variables include an indicator for the regulatory environment of the state where the bank is located ($d6$), which takes the value 1 if the institution is located in a unit-banking state and 0 otherwise. We also include the number of branches a bank operates ($d7$) and a series of indicators for institution type ($d9$–$d12$), which identify Federal Reserve nonmember commercial banks, savings and loans, mutual savings banks, and credit unions, respectively. Ferrier and Lovell (1990) included a variable for holding-company membership ($d8$), which was masked on the data tape we were able to obtain from the Federal Reserve and is therefore not included in our model.

Several variables are included in the vector, Z, to allow for heteroscedasticity in the one-sided error. These variables include the five measures of output (y_1–y_5) mentioned earlier, the number of demand deposit accounts, the number of time deposit accounts, the number of real-estate loans, the number of installment loans, and the number of commercial loans. In addition to the number of loans and deposits, we include the average dollar volume of each of these loans and deposits ($d1$–$d5$). The vector, Z, also includes the dummy ($d6$) indicating if the state in which the bank resides is a unit-banking state, and the variable ($d7$) indicating the number of branches a bank operates.

Our final data set differs from that used by Ferrier and Lovell in that they reported having 575 institutions in their data set but we have 555, 20 fewer. This discrepancy is apparently due to differences in the way we screened the institutions on the raw data tape, which contained 648 institutions. Because precise independent replication of empirical work is quite difficult, the reasonably close correspondence of our results is encouraging, although not necessary to illustrate the usefulness of our technique. Our data are available on request from the *JBES* editorial office.

4. ESTIMATION RESULTS

Due to the large number of parameters estimated in the cost function, the results are contained in Tables 1 and 2. The regression coefficients are presented in Table 1, and the variance parameters are given in Table 2. The second column of Table 1 contains the results from estimation by OLS. Nearly all of the coefficients have the expected sign and several are statistically significant. The R^2 of .992 indicates a very good fit, which is typical in the estimation of translog multiproduct cost functions. Column 3 of Table 1 contains the results from estimating the usual homoscedastic frontier model. As expected, the results are very similar to the OLS results. The intercept of the frontier model is, unsurprisingly, lower than that obtained by OLS. Olson, Schmidt, and Waldman (1980) showed that in the case of a cost function OLS estimates of all parameters except the intercept are unbiased and frontier estimation has the effect of lowering the intercept in the estimation of a cost function. Column 4 of Table 1 contains the results of estimating a heteroscedastic frontier model. The heteroscedastic frontier parameter estimates are about evenly split between those that are larger than the OLS or regular frontier parameter estimates and those that are smaller. In the multiproduct case, the estimated cost function is *twisted* by the heteroscedasticity. It is clear that, unlike in the homoscedastic case, the estimates from the heteroscedastic model are not simply OLS estimates with a lower intercept.

The estimates of the variance parameters are contained in Table 2. The statistical evidence for the existence of a homoscedastic frontier is weak. Our estimate for λ is .4976, and, using the Slutsky theorem, the approximate standard

Table 1. Regression Parameter Estimates

Variable	OLS	HOHN	HTHN	HOEX
Intercept	−7.011	−7.014	−7.177	−7.039
	(.189)	(.196)	(.241)	(.184)
$\ln y_i$	.287	.287	.226	.285
	(.024)	(.026)	(.029)	(.025)
$\ln y_2$	.539	.542	.650	.548
	(.025)	(.027)	(.034)	(.028)
$\ln y_3$	.046	.042	.029	.041
	(.015)	(.019)	(.021)	(.019)
$\ln y_4$	.033	.032	.034	.034
	(.016)	(.019)	(.019)	(.018)
$\ln y_5$	.066	.066	.051	.064
	(.012)	(.013)	(.014)	(.013)
$\ln w_1$	.377	.376	.324	.365
	(.028)	(.031)	(.029)	(.030)
$\ln w_2$	.387	.389	.416	.397
	(.027)	(.030)	(.029)	(.029)
$(\ln y_1)^2$	.068	.067	.036	.062
	(.024)	(.023)	(.024)	(.023)
$\ln y_1 \cdot \ln y_2$	−.035	−.033	−.011	−.029
	(.032)	(.030)	(.033)	(.029)
$\ln y_1 \cdot \ln y_3$	−.066	−.068	−.052	−.066
	(.018)	(.018)	(.021)	(.018)
$\ln y_1 \cdot \ln y_4$	.015	.016	.030	.020
	(.018)	(.017)	(.019)	(.017)
$\ln y_1 \cdot \ln y_5$	.035	.035	.025	.032
	(.015)	(.013)	(.015)	(.013)
$(\ln y_2)^2$	.128	.127	.102	.128
	(.039)	(.037)	(.045)	(.037)
$\ln y_2 \cdot \ln y_3$	.004	.005	.003	.003
	(.016)	(.018)	(.021)	(.018)
$\ln y_2 \cdot \ln y_4$	−.067	−.069	−.069	−.073
	(.021)	(.019)	(.022)	(.019)
$\ln y_2 \cdot \ln y_5$	−.047	−.047	−.045	−.044
	(.015)	(.014)	(.018)	(.013)
$(\ln y_3)^2$	.017	.016	.016	.017
	(.006)	(.010)	(.010)	(.010)
$\ln y_3 \cdot \ln y_4$	.017	.018	.015	.019
	(.013)	(.014)	(.014)	(.014)
$\ln y_3 \cdot \ln y_5$	.021	.021	.018	.021
	(.008)	(.009)	(.009)	(.009)
$(\ln y_4)^2$	.020	.020	.001	.019
	(.013)	(.017)	(.018)	(.017)
$\ln y_4 \cdot \ln y_5$	.002	.001	.005	.001
	(.010)	(.010)	(.012)	(.010)
$(\ln y_5)^2$	.004	.004	.004	.005
	(.007)	(.008)	(.008)	(.008)
$(\ln w_1)^2$	.103	.106	.110	.116
	(.055)	(.054)	(.054)	(.052)
$\ln w_1 \cdot \ln w_2$	−.055	−.057	−.053	−.065
	(.036)	(.043)	(.041)	(.041)
$(\ln w_2)^2$	.167	.168	.174	.176
	(.039)	(.051)	(.048)	(.049)
$\ln y_1 \cdot \ln w_1$	.018	.018	.029	.018
	(.038)	(.037)	(.038)	(.035)
$(\ln y_1 \cdot \ln w_2$	−.046	−.045	−.033	−.044
	(.036)	(.036)	(.036)	(.036)
$\ln y_2 \cdot \ln w_1$	.055	.057	.019	.057
	(.037)	(.037)	(.035)	(.036)
$\ln y_2 \cdot \ln w_2$	−.002	−.004	−.003	−.003
	(.034)	(.037)	(.036)	(.035)
$\ln y_3 \cdot \ln w_1$	−.046	−.046	−.043	−.048
	(.018)	(.019)	(.020)	(.019)
$\ln y_3 \cdot \ln w_2$	.015	.016	.013	.019
	(.021)	(.023)	(.238)	(.022)
$\ln y_4 \cdot \ln w_1$	−.005	−.005	.005	−.003
	(.025)	(.027)	(.028)	(.027)
$\ln y_4 \cdot \ln w_2$	.038	.039	.039	.037
	(.021)	(.024)	(.025)	(.024)
$\ln y_5 \cdot \ln w_1$	.002	.002	−.013	−.000
	(.016)	(.017)	(.017)	(.017)
$\ln y_5 \cdot \ln w_2$	−.012	−.014	−.010	−.015
	(.015)	(.018)	(.016)	(.018)
$\ln d_1$	.314	.315	.220	.307
	(.015)	(.014)	(.018)	(.013)
$\ln d_2$	.435	.439	.591	.435
	(.022)	(.020)	(.031)	(.021)
$\ln d_3$	.028	.019	.016	.020
	(.015)	(.012)	(.018)	(.012)
$\ln d_4$	.005	.005	.011	.005
	(.012)	(.012)	(.016)	(.012)
$\ln d_5$	.059	.060	.025	.057
	(.010)	(.010)	(.012)	(.010)
d_6	−.015	−.015	−.044	−.015
	(.013)	(.014)	(.020)	(.014)
d_7	.002	.002	.002	.002
	(.001)	(.001)	(.001)	(.001)
d_9	.006	.006	.010	.007
	(.012)	(.013)	(.011)	(.012)
d_{10}	.064	.070	−.016	.062
	(.047)	(.055)	(.048)	(.054)
d_{11}	.116	.121	.011	.114
	(.033)	(.035)	(.033)	(.034)
d_{12}	.075	.078	.000	.072
	(.067)	(.129)	(0.086)	(.117)
R^2	.992	—	—	—

NOTE: Figures in parentheses are standard errors.

error is .8803. This suggests that there is no (homoscedastic) frontier. This test of significance is invalid, however, because the value of λ under the null hypothesis of no inefficiency ($\lambda = 0$) is on the boundary of the admissible parameter space. A valid test based on the coefficient of skewness, b_1, was given by Schmidt and Lin (1984), where $b_1 = (\Sigma e_i^3/N)/(\Sigma e_i^2/N)^{1.5}$. This yields a test statistic of .0237, which does not exceed the critical value of .243 for $\alpha = .01$ given in *Biometrika Tables for Statisticians* (Vol. 1, table 34B). Alternatively, we perform a test of symmetry given by Pagan and Hall (1983). This test is based on a statistic, h, which follows the standard normal distribution, where $h = [(N^* b_1)/6]^{.5}$. Our value of h is .2275, which is not significant at any of the usual levels of significance. The results of all of these tests indicate that no homoscedastic frontier is present, but we do find compelling evidence for the existence of a *heteroscedastic* frontier.

Column 4 of Table 2 gives the estimates of parameters in the vector γ, and these parameters provide information on the presence of heteroscedasticity. Four of these coefficients are significantly different from 0. Although this is evidence of the presence of heteroscedasticity, as we noted earlier a likelihood ratio test is possible. The value of the chi-squared statistic is 98.4, which exceeds the critical value of χ^2_{12} at any of the usual levels of significance. This provides further evidence suggesting the presence of heteroscedasticity.

Table 2. Variance Parameter Estimates

Variable	OLS	HOHN	HTHN	HOEX
Intercept (γ_0)	—	−3.055[a]	−4.723	3.154
		(1.595)	(4.211)	(.243)
$\ln y_1(\gamma_1)$	—	—	1.100	—
		—	(.515)	
$\ln y_2(\gamma_2)$	—	—	−1.242	—
		—	(.535)	
$\ln y_3(\gamma_3)$	—	—	.133	—
		—	(.267)	
$\ln y_4(\gamma_4)$	—	—	−.112	—
		—	(.243)	
$\ln y_5(\gamma_5)$	—	—	−.034	—
		—	(.272)	
$\ln d_1(\gamma_6)$	—	—	1.145	—
		—	(.481)	
$\ln d_2(\gamma_7)$	—	—	−1.392	—
		—	(.552)	
$\ln d_3(\gamma_8)$	—	—	.029	—
		—	(.279)	
$\ln d_4(\gamma_9)$	—	—	.007	—
		—	(.331)	
$\ln d_5(\gamma_{10})$	—	—	.371	—
		—	(.278)	
$d_6(\gamma_{11})$	—	—	.573	—
		—	(.452)	
$d_7(\gamma_{12})$	—	—	.007	—
		—	(.037)	
θ	—	−2.357[b]	−2.551	2.418
		(.179)	(.065)	(.054)

NOTE: Figures in parentheses are standard errors.
[a] $\sigma_v = \exp(\gamma_0)$.
[b] $\sigma_w = \exp(\theta)$.

The two inefficiency measures discussed previously were calculated for each bank in the data set. For the homoscedastic frontier model (HOHN) with the half-normal error specification, the mean inefficiency was 3.76% with a standard deviation of .84%. In the heteroscedastic frontier model (HTHN), the mean inefficiency score rose to 5.54%, with a standard deviation of 8.51%. In absolute terms this increase in mean inefficiency is modest, but proportionately it is nearly 50% higher. The minimum inefficiency score in the HOHN model was 1.82%, and the maximum score was 8.03%; the minimum score in the HTHN model was lower at .05%, and the maximum was much higher at 65.48%. Correcting for heteroscedasticity has this effect because of the twisting of the cost frontier as noted earlier. Not accounting for heteroscedasticity leads one to overestimate inefficiency for small firms and underestimate inefficiency for large firms.

In addition to the absolute measures obtained from the estimations, the relations between the firm-specific inefficiency rankings are of interest to determine the extent to which correcting for heteroscedasticity affects the determination of relative rather than absolute inefficiency. One measure of this relation is the correlation between the two rankings of firms by their inefficiency measures in the homoscedastic and heteroscedastic cases. This correlation was .2018, which was statistically significantly different from 0 at an α-level of .01.

The firm-specific inefficiency measures and the firm rankings they make possible may be as sensitive to alternative error specifications as they are to the correction for heteroscedasticity. To investigate this issue, a homoscedastic frontier model with an exponential one-sided error term (HOEX) was estimated for comparison with the HOHN model. The estimation results for the HOEX model are given in Column 5 of Table 1. The estimation results are very similar to the HOHN specification. The mean inefficiency score for HOEX specification was 4.27% with a standard deviation of 2.20%; this mean was roughly 13% higher than the HOHN specification. The minimum score for the HOEX model was 1.48%, relatively close to the HOHN model, but the minimum score for the HOEX model at 23.52% was about three times that of the HOHN model. The correlation between the inefficiency rankings obtained from HOHN and HOEX models was .9987, indicating a very close correspondence between the two rankings. Moreover, the correlation between the HOEX rankings and the HTHN rankings, at .2233, was quite similar to the correlation of .2018 between the HOHN and HTHN rankings. Thus the firm-specific inefficiency rankings obtained from our frontier-model estimates appear much more sensitive to the correction for heteroscedasticity than to these alternative specifications of the error term.

5. CONCLUSIONS

Frontier estimation has been widely used in economics to estimate firm-specific inefficiency. Because most of the measures of inefficiency are based on residuals, it is critically important to note that the residuals and consequently the inefficiency measures may be distorted by specification errors. This article discusses and develops the estimation of a heteroscedastic frontier model by maximum likelihood. The frontier cost model is estimated under assumptions of homoscedasticity and heteroscedasticity for the one-sided error. Both models are estimated using bank cost data. A likelihood ratio test rejects the null hypothesis of homoscedasticity. The estimated coefficients in the models are different, but the most dramatic difference is found in the firm-specific inefficiency measures from each model. When heteroscedasticity is incorporated into the estimation, average inefficiency estimates are about 50% higher at the mean. Furthermore, the ranking of firms as to their relative inefficiency changes dramatically when the correction for heteroscedasticity is incorporated into the estimation, much more than the rankings do under different specifications for the one-sided error term when heteroscedasticity is ignored. This is considerable evidence that inefficiency measures are sensitive to specification errors like heteroscedasticity and must be viewed with caution unless the heteroscedasticity is incorporated into the estimation.

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